Metric theories of gravity for galactic dynamics and gravitational lenses.

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Talk available in
http://www.mendozza.org/sergio/modifiedgravity

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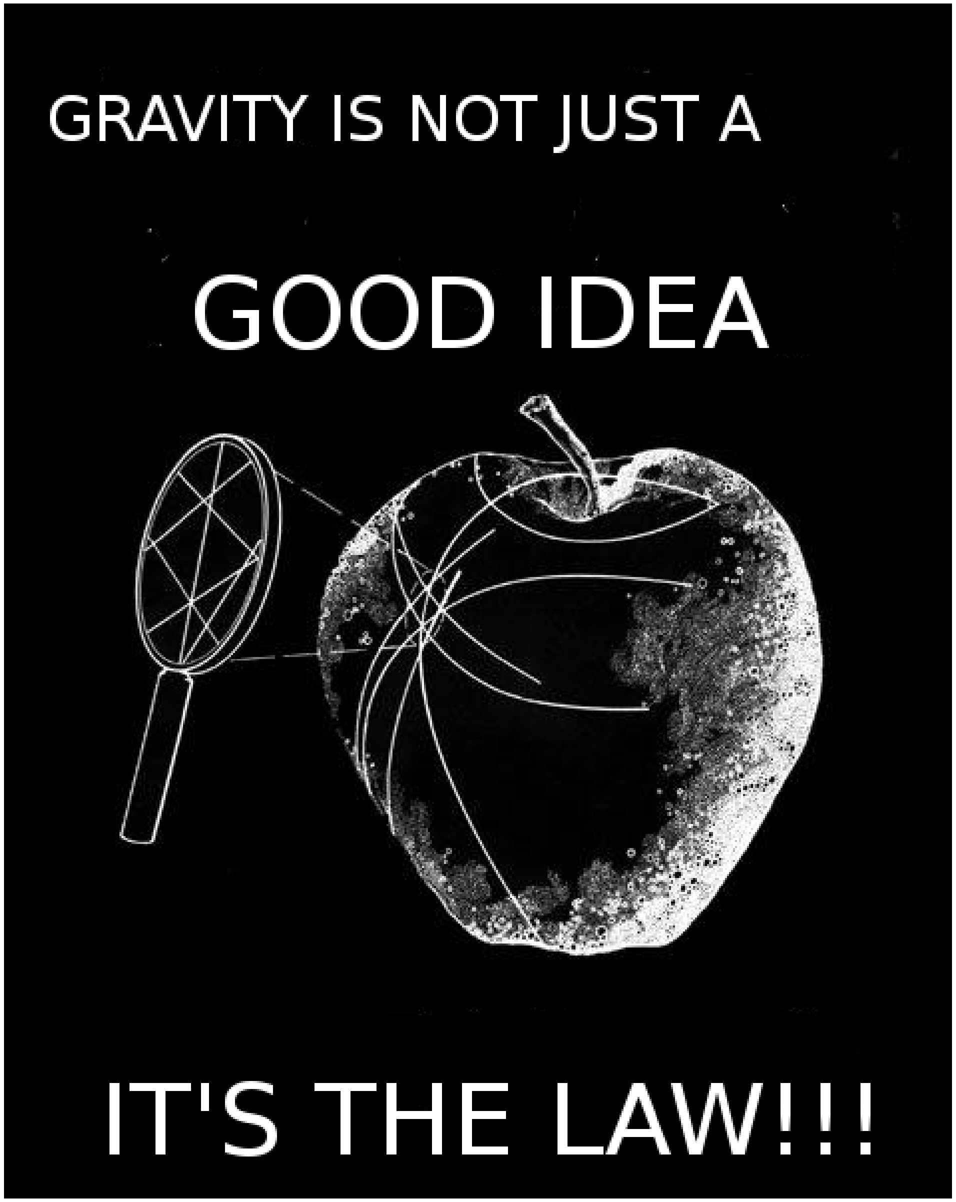
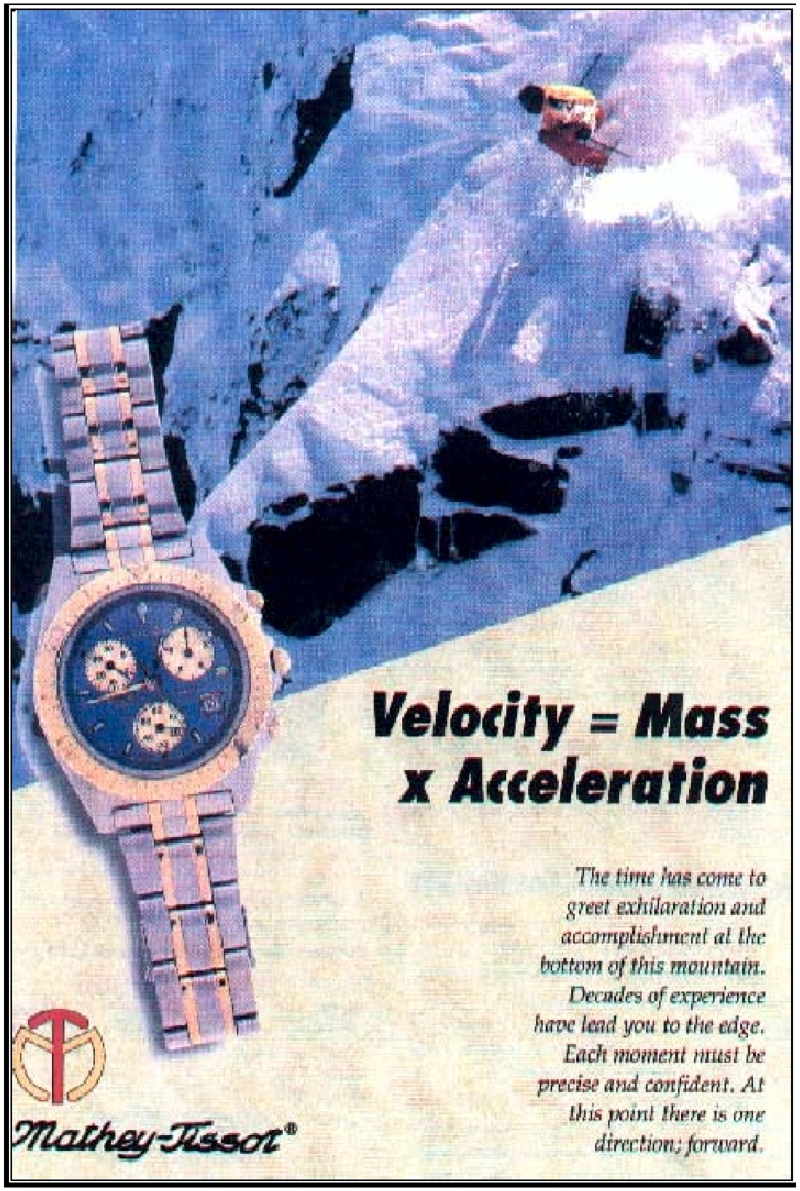
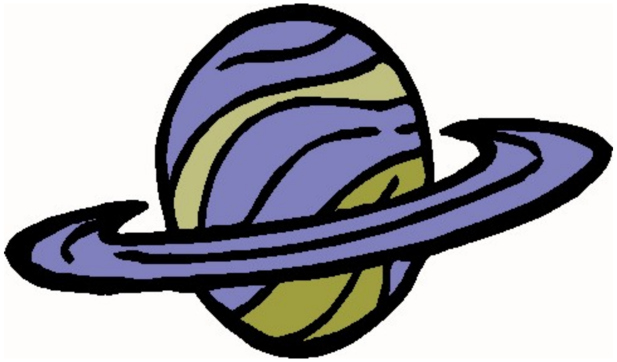

GRAVITY IS NOT JUST A
GOOD IDEA
IT'S THE LAW!!!

☆ Is there something wrong with modifying the standard laws of physics?



1 Introduction

- ☆ Astronomers call *dark* anything they cannot comprehend. E.g. dark matter and dark energy.
- ☆ Neptune was a dark matter candidate on his time. Bouvard (1821), Adams (1843) & Airy (and independently Le Verrier -1846) postulated its existence due to anomalous movement of Uranus.
- ☆ Le Verrier (1900's) proposed that the motion of the perihelium of mercury was produced by an unknown planet: *Vulcan*, which was never found.
- ☆ Einstein (1915) calculated that mercury's precession was a natural consequence of his modification to the Universal law of gravitation.



Dark matter candidates at the beginning of the 20th century

2 Are all the laws of physics valid at all scales?

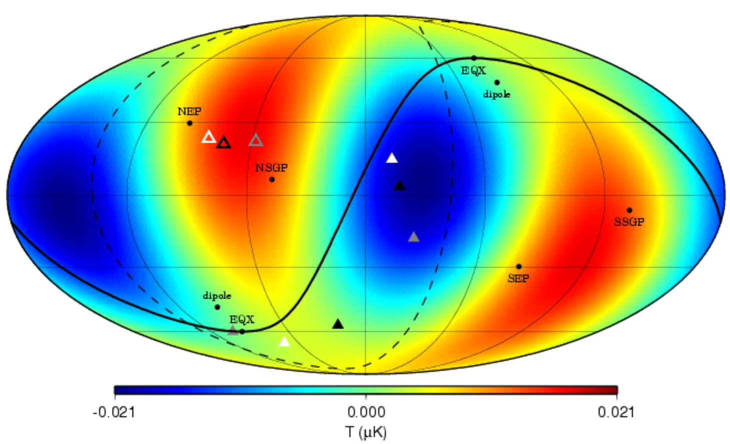
- In 1913, Bohr proposed his atom postulates to explain atomic spectra.
- We all know now that classical physics is not valid at atomic scales.

☆ General relativity tests have been made at scales less than the solar system scale ($\sim 10^{11}$m).

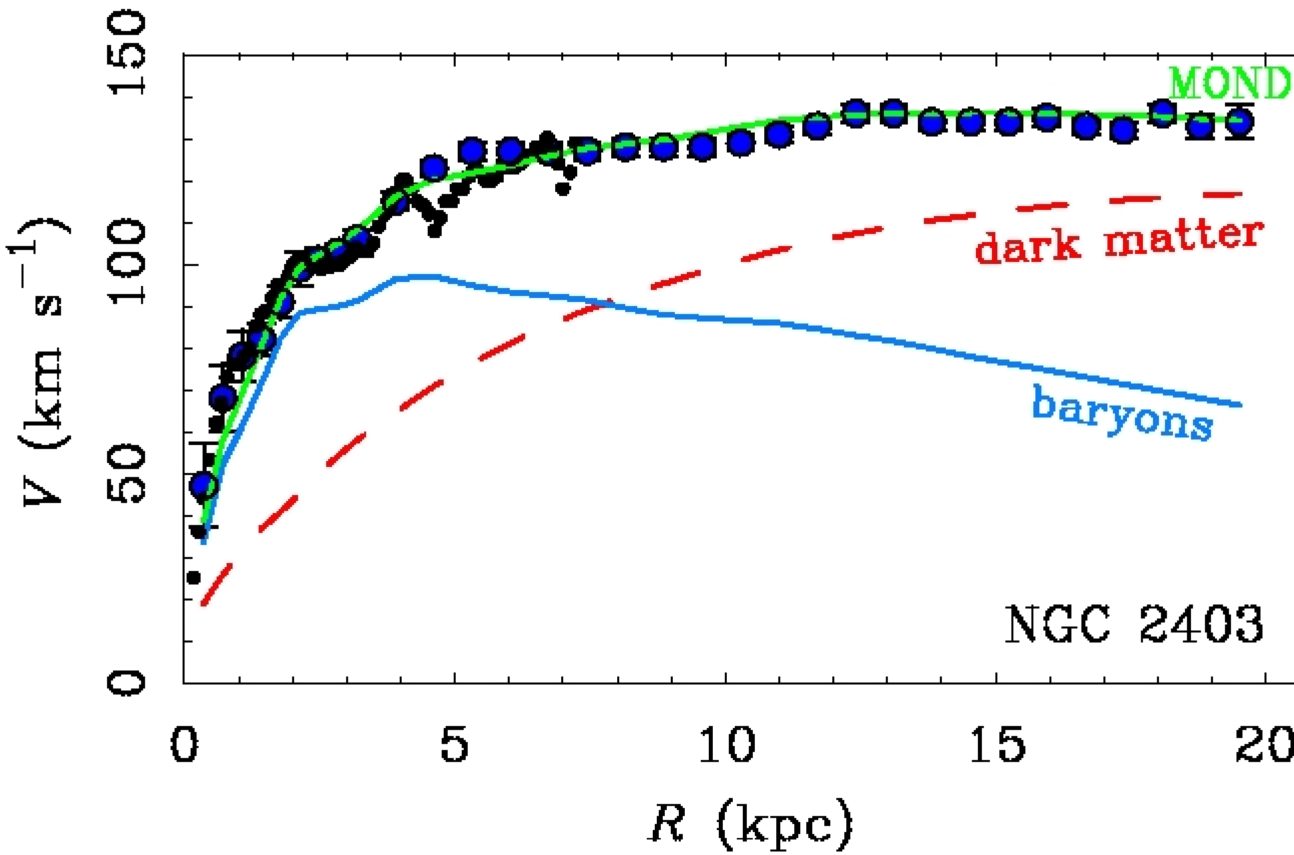
☆ Newtonian mechanics is not valid in scales of the order of ten to fifteen orders of magnitude below our standard scales (size of the Hydrogen atom).

☆ Is it possible that at scales larger than ($\sim 10^{20}$m, galactic scales) solar system sizes gravity needs a modification?

☆ *Note that measurements of e.g. WMAP do not measure* $\Omega_\Lambda, \Omega_{DM}, \Omega_{DE}$ etc. This quantities are part of a standard cosmological model assuming Einstein's gravity to be valid. When the mathematical model is applied to WMAP observations, then the corresponding energy densities are fixed.

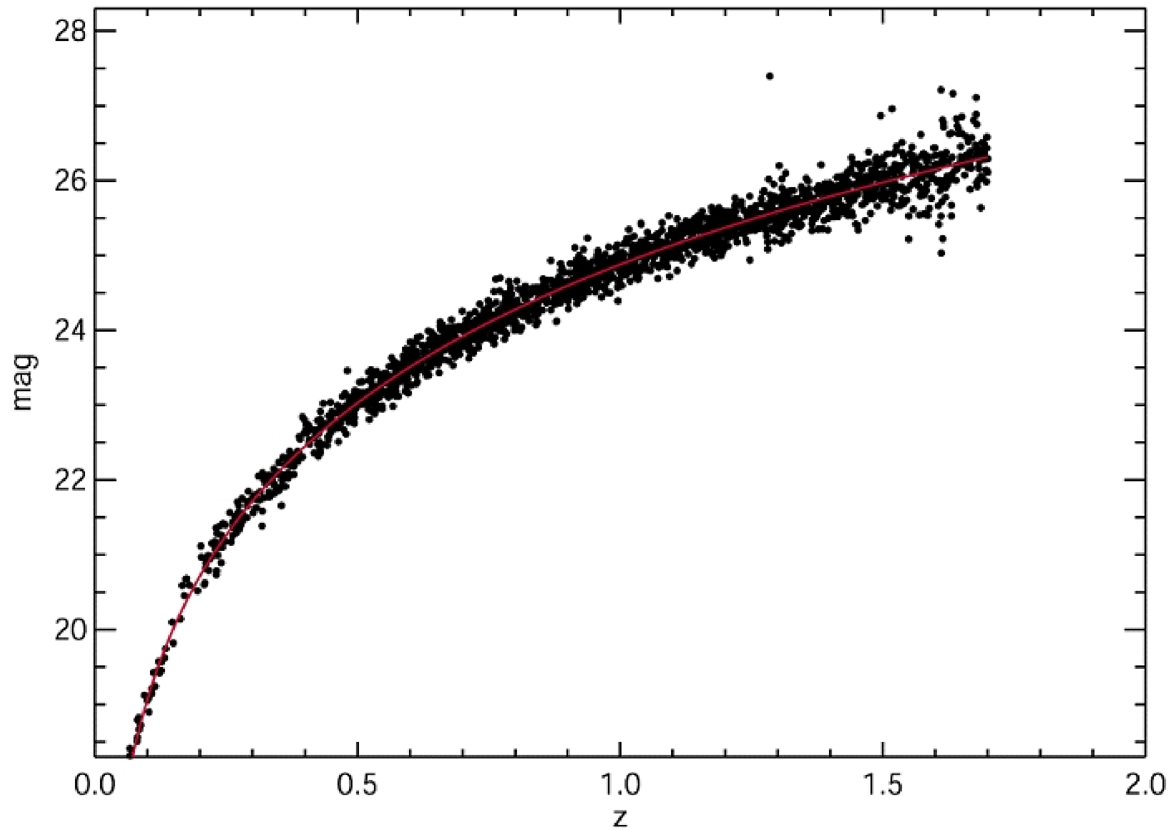


3 Dark matter



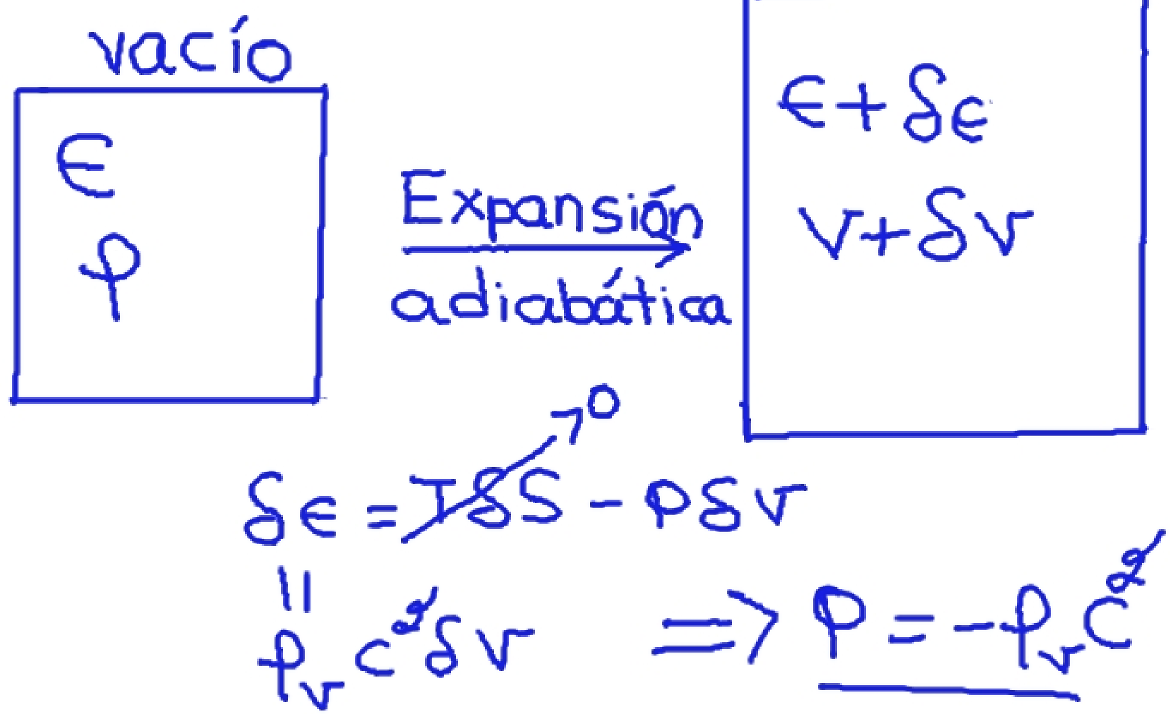
Consequences: more matter (cold and dark), scalar fields or gravity modifications.

4 Accelerated universe today! (in the past: inflation)



Consequences: ¡A repulsive force! : cosmological constant, scalar fields, quintessence or modifications to gravity.

- ☆ Early universe requires an inflationary epoch: causality and flatness.
- ☆ Solution: cosmological constant ⇒ scalar fields are produced by a cosmological vacuum: negative pressure.



5 General relativity

- Built with a metric $g_{\mu\nu}$ and a 4th dimensional manifold M (space-time): (M,g)
- Hilbert–Einstein action:

$$S = -mc\int \mathrm{d}s - \frac{1}{16\pi G}\int (R+2\Lambda)\sqrt{-g}\,\mathrm{d}^4x + \int L_{\mathrm{m}}\sqrt{-g}\,\mathrm{d}^4x. \tag{1}$$

- The problem with (1) is that R is not the only possibility. ¡¡It can be changed by $f(R)$ or contractions of Riemann's tensor that produce a scalar! ! !
- Variations of (1) produce:

$$\frac{\mathrm{d}^2x^\mu}{\mathrm{d}\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau} = 0, \qquad G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} + \Lambda g_{\mu\nu}. \tag{2}$$

- Note: Einstein made: $G_{\mu\nu} \propto T_{\mu\nu}$. This relation has been corroborated experimentally. E.g. Eddington & solar eclipse, mercury's perihelium, pulsar's decay, gravitational redshift... in scales $\lesssim$ solar system!

6 Modifications to gravity

v · d · e

Theories of gravitation

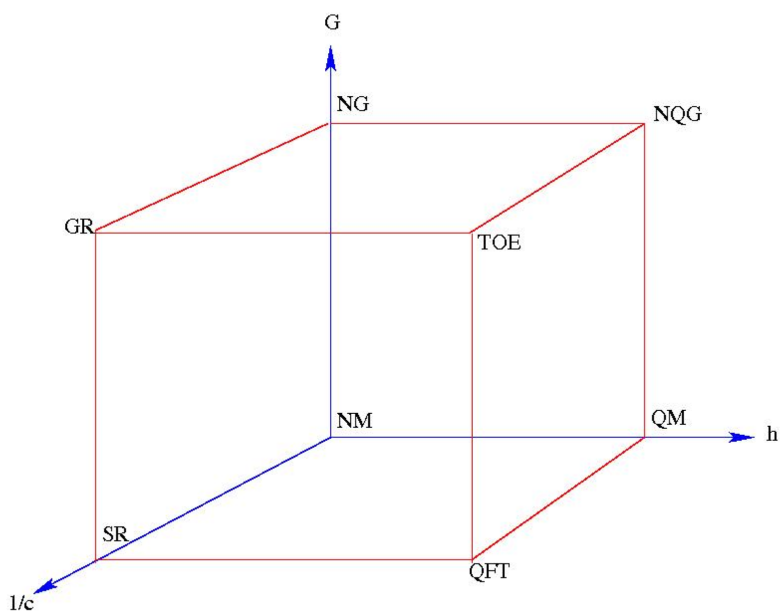
Standard	Alternatives to GR	Unified field theories	Other

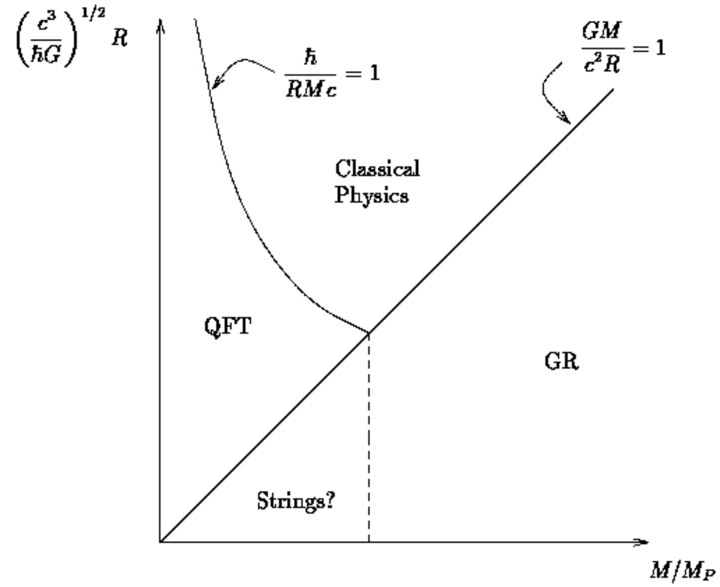
▪ **History of gravitational theory**
▪ **Newtonian gravity (NG)**
  ▪ Classical mechanics
▪ **General relativity (GR)**
  ▪ History
  ▪ Mathematics
  ▪ Resources
  ▪ Tests
  ▪ Twistors [S]
	▪ Classical theories of gravitation [S]
	▪ Conformal gravity [S]
	▪ **Scalar theories**
	  ▪ Nordström
	  ▪ Yilmaz
	▪ **Scalar-tensor theories** [S]
	  ▪ Brans-Dicke
	  ▪ Self-creation cosmology
	▪ **Bimetric theories** [S]
	▪ **Other alternatives**
	  ▪ Einstein-Cartan
	    ▪ Cartan connection
	  ▪ Whitehead [S]
	  ▪ Nonsymmetric gravitation [S]
	  ▪ Scalar-tensor-vector [S]
	  ▪ Tensor-vector-scalar [S]
		▪ Teleparallelism
		▪ Geometrodynamics
		▪ **Quantum gravity**
		  ▪ Semiclassical gravity
		  ▪ Discrete Lorentzian QG [S]
		  ▪ Euclidean QG [S]
		  ▪ Induced gravity [S]
		  ▪ Loop quantum gravity
		  ▪ Wheeler-deWitt eqn
		▪ **Theory of everything**
		  ▪ Supergravity
		  ▪ M-theory
		  ▪ Superstrings
		  ▪ String theory
		    ▪ String theory topics
			▪ **Higher-dimensional GR**
			  ▪ Kaluza-Klein
			  ▪ DGP model [S]
			▪ **Alternatives to NG**
			  ▪ Aristotle
			  ▪ Fatio-Le Sage
			  ▪ MOND
			▪ **Unclassified**
			  ▪ Composite gravity [S]
			  ▪ Massive gravity [S]

[S] = Stub only

Source: wikipedia.org

- Einstein modified his theory interested in unification, taking ideas of Kaluza and Klein.
- The necessity of modifying Einstein's gravity came with ideas of Eddington and Dirac.
- Fundamental constants of physics: $G,\ e,\ \ \hbar,\ c,$.
- Their numerical value fits perfectly nuclear reactions, structure formation in the universe, etc.
- Okun represented the importance of these constants using a three dimensional cube, the so called Okun's cube:





- ☆ Take: G, e, $\hbar$, c, $m_{\rm prot}$.
- ☆ The value of these dimensional quantities depends critically of the system of units used. Experimentally, one chooses always dimensionless quantities to avoid dependencies on the system of units chosen.
- ☆ For example, Stoney (1874) built:

$$m_{\rm J} = \left(e^2/G\right)^{1/2}, \quad l_{\rm J} = \left(Ge^2/c^4\right)^{1/2}, \quad t_{\rm J} = \left(Ge^2/c^6\right). \tag{3}$$

- ☆ Another example, the good one is the Natural units of Planck (1899):

$$m_{\rm P} = (hc/G)^{1/2}, \quad l_{\rm P} = \left(Gh/c^3\right)^{1/2}, \quad t_{\rm P} = \left(Gh/c^5\right). \tag{4}$$

- ☆ Dimensionally, using G, e, $\hbar$, c, $m_{\rm prot}$ its possible to built two dimensionless parameters:

$$\alpha := \frac{e^2}{\hbar c} = 0.007297352568(24) \approx \frac{1}{137.03599911(46)}, \tag{5}$$

$$\alpha_G := \frac{G m_{\rm prot}^2}{\hbar c} \approx 10^{-38}. \tag{6}$$

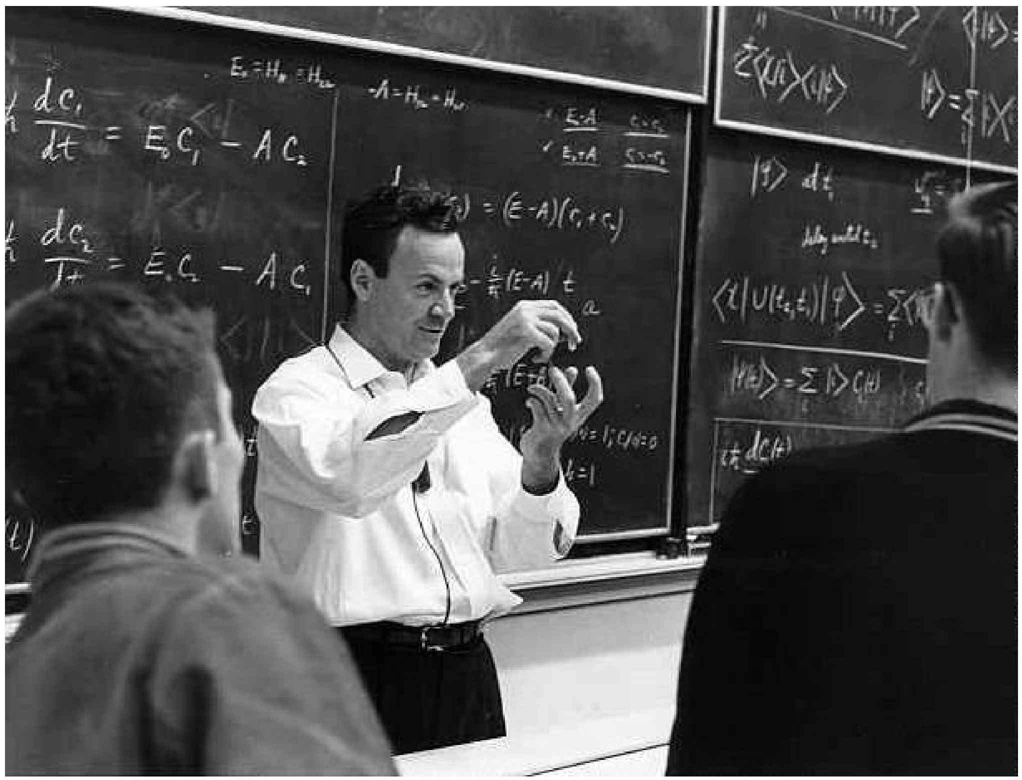
- ☆ α can be thought of the ratio of two energies:

$$\alpha = \left.\frac{e^2/r}{hc/\lambda}\right|_{\lambda=s}. \tag{7}$$

- ☆ α is the coupling constant between the proton and the photon in quantum electrodynamics.
- ☆ Value of α CAN'T be calculated theoretically.
- ☆ α is one of the 20 external parameters needed for the standard model of particle physics.
- ☆ Antropic arguments (Dicke): α must be well tuned. If α changes by say 4% then carbon can't be produced in the interior of stars. Even if α would change by a decimal value, stellar fusion would not occur.
- ☆ Feynman: "…one of the greatests mysteries of physics: a magic number that appears in nature and can't be understood by humans".
- ☆ Mathematically, with great precision (note that 29 y 137 are prime numbers, on the place 10 and 33 respectively):

$$\alpha = \frac{\cos(\pi/137)}{137}\frac{\tan(\pi/137\times 29)}{\pi/(137\times 29)} \approx \frac{1}{137.0359997867}. \tag{8}$$

- ☆ Physically, α has been calculated approximately using statistical mechanics.
- ☆ Feynman: "...from its discovery, it has been a mystery and all respectful theoretical physicist writes down this number and sticks it on its wall and worries about its meaning!!! "



7 Large number hypothesis: Dirac (1938).

☆ Atomic scales come from dimensional combinations of e, c, $m_{\rm a}$ (e.g. proton mass) and $\hbar$ $\Rightarrow$

$$R_{\rm a} = \frac{e^2}{c^2 m_{\rm a}}, \qquad T_{\rm a} = \frac{R_{\rm a}}{c}.$$

☆ Time ($T_{\rm U}$) y size ($R_{\rm U} = cT_{\rm U}$) of the Universe with a mass $M_{\rm U}$, all together with a gravitational constant G.

☆ These relations satisfy (in terms of dimensionless numbers):

$$\alpha := e^2/\hbar c \approx 1/137 \approx 10^0,$$
$$\frac{\text{Universe size}}{\text{proton size}} = \frac{cT_{\rm U}}{R_{\rm a}} \approx 10^{40},$$
$$\frac{\text{electric force}}{\text{gravitacional force}} = \frac{e^2}{Gm_a^2} \approx 10^{40}. \qquad (9)$$

$$\Longrightarrow G \propto \frac{1}{T_{\rm U}}. \qquad (10)$$

8 α variations?

- Difficult to know which one varies: G, $\hbar$, e. Dirac assumed G. Gamow postulated e. To date nobody has postulated temporal variations on $\hbar$ but people has done work on $m_{\rm prot}$ variations.
- Curiously, variations on the physical constants of nature were first done by Kelvin ($c(t)$). Weyl and afterwards, Eddington proposed the large number hypothesis, used greatly by Dirac.
- Teller (1948) showed that if $G \propto 1/T_{\rm U}$ then the earth's temperature on the past (pre–Cambrian epoch) would had been so elevated that life could not been sustained.
- Gamow suggested $e \propto 1/T_{\rm U}$, but its idea was quickly abandoned since it contradicted geophysical data and radioactive decay experiments.
- Dicke (1956) showed that the coincidence between the large numbers was because the age of the universe (i.e. time when we do our astronomical observations) $\sim$ time spent by a star on the main sequence. $\Longrightarrow$ This time is then necessary in order to make observers made of elements heavier than H & He !!!

- ☆ These antropic arguments made by Dicke were not the only reason to conclude no–variations. He also repeated Eötvos experiment to determine with a greater precision the value of G.
- ☆ Physical reasons to have variations of the fundamental constants:
  - ☆ Better candidates for unification (quantum theories of gravity) exist in greater dimensions $\Rightarrow$ the shadows of the constants may vary easily when they vary on greater dimensions.
  - ☆ Most probably, the value of the physical quantities come from a spontaneous symmetry breaking (e.g. chaotic inflation).
  - ☆ The strongest one is the experiment. Webb et al. (2001):

$$\Delta\alpha/\alpha = (-0.57 \pm 0.10) \times 10^{-5}, \tag{11}$$

for a sample of quasars in the interval $1 \lesssim z \lesssim 3.5$.

- ☆ In the 1960's Jordan–Brans–Dicke completed their theory to solve the problem $G(t)$. This theory takes into account Mach's principle. This theory gives the foundations for what its known as scalar–tensor $(\phi, g_{\mu\nu})$ theories of gravity.
- ☆ For many decades scalar–tensor theories were used as the main way to understand gravity. Even Milgrom and Beckenstein tried to built a Relativistic Theory of MOND (RAQUAL) in the 1990's in order to account for relativistic phenomena similar to MOND.
- ☆ Beckenstein (2004) used a Tensor–Vector–Scalar (TeVeS) theory of gravity in order to built a relativistic theory of MOND.
- ☆ Metric $f(R)$ theories are very natural generalizations of Einstein's gravity.
- ☆ Action given by

$$S = -m \int \mathrm{d}s - \int f(R) \sqrt{-g} \, \mathrm{d}^4 x + \int L_{\mathrm{m}} \sqrt{-g} \, \mathrm{d}^4 x. \tag{12}$$

- ☆ Variations of (12) produce

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau} \frac{\mathrm{d}x^\beta}{\mathrm{d}\tau} = 0, \qquad G_{\mu\nu} = T^{(\mathrm{mat})}_{\mu\nu} + T^{(\mathrm{curv})}_{\mu\nu}, \tag{13}$$

where $T_{\mu\nu}^{(\mathrm{mat})} := T_{\mu\nu}^{(\mathrm{mat\text{-}std})}/f'(R)$ and $T_{\mu\nu}^{(\mathrm{curv})}$ depends on $f(R)$ and its derivatives up to 4th order.

9 Newtonian Theory Modifications (MOND)

☆ Milgrom (1980's) propose that in order to explain rotation curves in spiral galaxies there is no need for dark matter, only a modification to Newton's Universal Gravitation:

$$\frac{\boldsymbol{a}^2}{a_0} = -G\frac{M}{r^2}. \tag{14}$$

with $a_0 \approx 10^{-8}\,\mathrm{cm/s^2} \approx cH_0$.

☆ The problems with momentum conservation are solved with a full theory: AQUAL (Aquadratic Lagrangian) (Bekenstein, Milgrom 1984):

$$L = -\frac{a_0^2}{8\pi G} f\left(\frac{|\nabla\phi|^2}{a_0^2}\right) - \rho\phi, \tag{15}$$

with $-\nabla\phi = \boldsymbol{a}$ and

$$f(y) = \begin{cases} y, & \text{Newton} \quad (a \gtrsim a_0) \\ y^{3/2}, & \text{MOND } (a \lesssim a_0). \end{cases} \tag{16}$$

☆ Field equations for AQUAL:

$$\nabla \cdot \left\{ f\left(\frac{|\nabla\phi|^2}{a_0^2}\right)\nabla\phi\right\} = 4\pi G\rho. \tag{17}$$

☆ Equation (17) coincides with MOND for systems with spherical symmetry and others.

☆ The problem is to make a relativistic theory that converges to AQUAL in the weak field limit.

☆ There is one solution: relativistic AQUAL (Bekenstein, Milgrom 1984) which introduces a scalar field ϕ and the field $g_{\mu\nu}$.

☆ The problem with relativistic AQUAL: waves are *acausal* and the theory does not produce gravitational lenses that bend light more than general relativity.

10 TeVeS (Bekenstein 2004)

- ☆ TeVeS (Tensor Vector Scalar): metric: $g_{\mu\nu}$, time-like vector: U^α and a scalar potencial ϕ.
- ☆ Action:

$$S = \frac{1}{16\pi G} \int \mathrm{d}^4 x \sqrt{-g}\, (R + L_U + L_\phi) \tag{18}$$

with

$$L_U = -\text{MANY HORRIBLE TERMS}$$
$$L_\phi = -\ \text{MANY MORE}$$

and all depend on an "empirical potential"

$$V(\eta) = \text{SOME MORE}, \tag{19}$$

plus necessary coupling constants, etc.

<u>¡It's certainly not something that occurs naturally!</u>

TeVeS advantages:

- ☆ Non–relativistic limit is in agreement with phenomenology of extragalactic data.
- ☆ Reproduces gravitational lensing observations.
- ☆ It has "harmony" with cosmological results.
- ☆ There exist repulsive solutions in this theory which can reproduce acoustic peaks of WMAP (cf. recent articles of L.M. Diaz-Rivera).

The main problem with TeVeS: IT'S TOO COMPLICATED.

11 But TeVeS is just the beginning...

- ☆ Metric theories of gravitation $f(R)$ come from a variational principle, so they assure conservation of energy and momentum.
- ☆ Metric theories of gravitation of the form $f(R) = R^n$ produce gravitacional waves propagating at the velocity of light.
- ☆ Some theories of gravity produce light deflections greater than general relativity under certain values of the parameters r_m/s and α relevants for different astronomical environments (e.g. galaxy clusters and galaxies).
- ☆ *WE DO NOT HAVE IDEA WHAT DARK MEANS!!!* We still have continue working on dark matter models and modifications of gravity ones.